A pair of novel statistics to improve constraints on primordial non-Gaussianity and cosmological parameters

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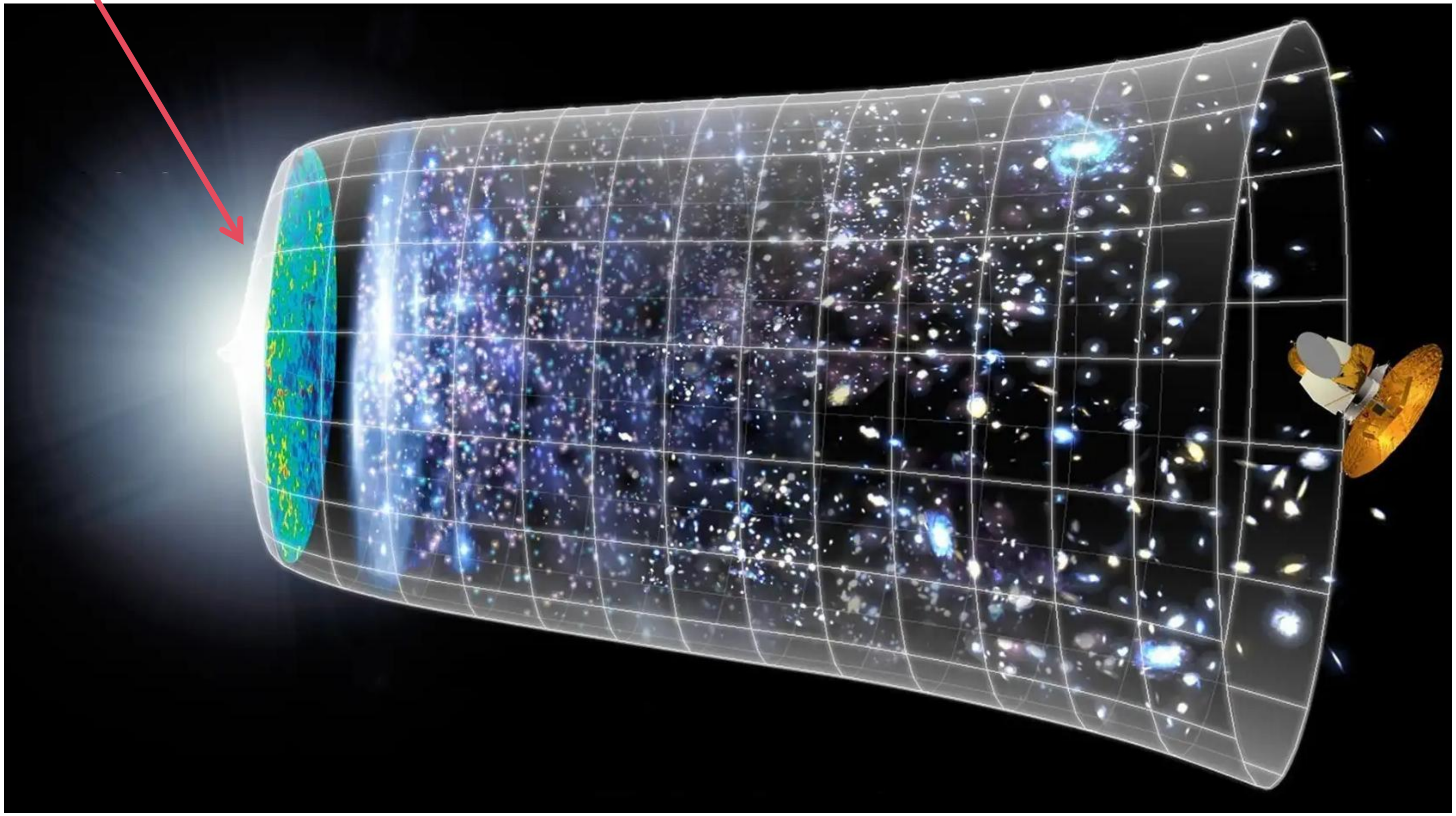
Outline

- What is primordial non-Gaussianity (PNG)?
- Why is PNG important?
- The challenge of constraining PNG
- Our novel statistics
- Parameter forecasts

What is Primordial non-Gaussianity (PNG)?

Cosmic inflation: the early Universe underwent a phase of accelerated expansion in which quantum fluctuations were stretched at cosmological scales.



What is Primordial non-Gaussianity (PNG)?

- There exists a broad diversity of inflationary models.
- All existing models predict tiny deviations from Gaussianity of primordial fluctuations, i.e. PNG.
- The primordial gravitational potential Φ:

$$\Phi = \Phi_{\mathrm{G}} + f_{\mathrm{NL}}^{\mathrm{X}}(\Phi_{\mathrm{G}}^2 - \langle\Phi_{\mathrm{G}}^2\rangle) + \text{higher-order terms}$$

'X' refers to local, equilateral, or orthogonal

- The bispectrum $B_\Phi(k_1, k_2, k_3)$ is the lowest order statistic sensitive to non-Gaussian features in the primordial potential field.

$$\langle\Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{\mathrm{D}}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B_\Phi(k_1, k_2, k_3)$$

- Local shape: $k_1 \ll k_2 \approx k_3$
- Equilateral shape: $k_1 \approx k_2 \approx k_3$
- orthogonal shape: $2k_1 \approx 2k_2 \approx k_3$ (negative amplitude)

  $k_1 \approx k_2 \approx k_3$ (positive amplitude)

Why is PNG important?

- Discriminating between various inflationary models.
- Providing clues about the high energy physics of the early Universe.
- Bridging the early Universe and late Universe.
- Indicating new physics beyond the standard model of cosmology.
- ......

The challenge of constraining PNG

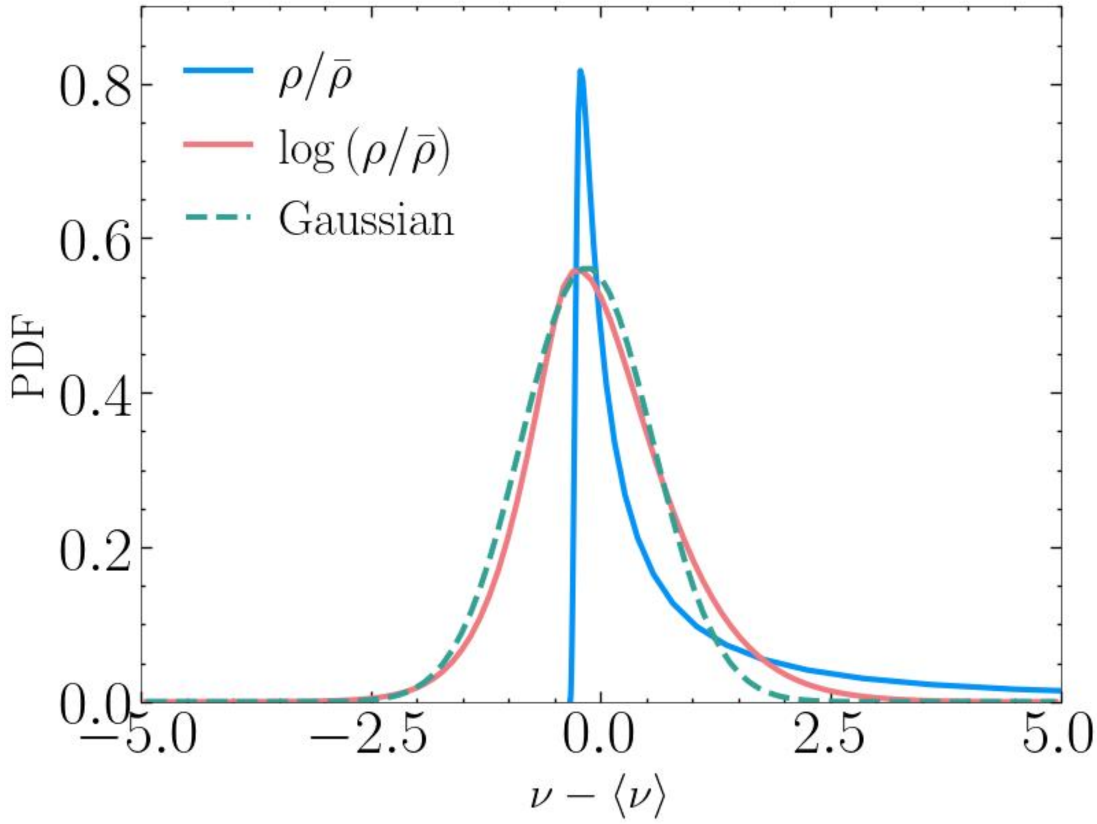
- The signal of PNG is faint.
- The most stringent constraints come from measurements of the cosmic microwave background (CMB) anisotropies by the Planck satellite:

$$f_{\rm NL}^{\rm local} = -0.9 \pm 5.1, \quad f_{\rm NL}^{\rm equil} = -26 \pm 47, \quad f_{\rm NL}^{\rm ortho} = -38 \pm 24$$

  - Obstacles: 2D, diffusion damping at small scales
- The current and future large-scale structure (LSS) surveys hold promise for offering enhanced sensitivity to PNG
  - Advantages: they can map a huge 3D volume of the Universe
  - Challenge: the late-time non-Gaussianity
- Advanced methods:
  - marked power spectrum, power spectra in cosmic web environments, one-point probility distribution, neural network, persistent homology, ...

Our novel statistics

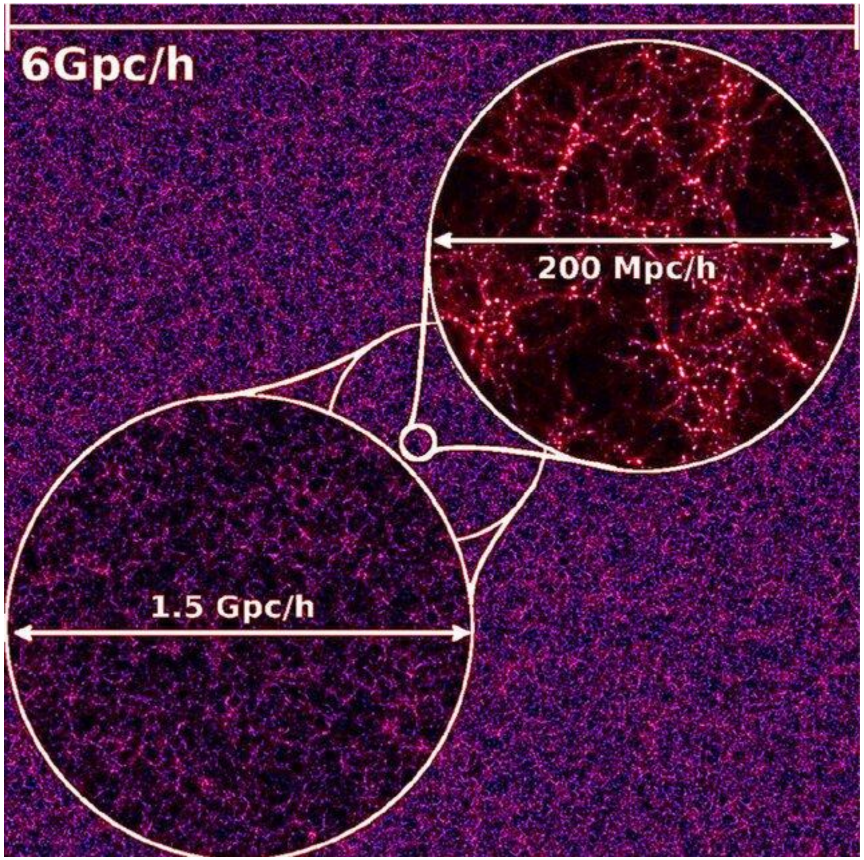
- The crucial features of the late-time matter distribution:
  - the PDF of density field is nearly log-normal (Hamilton 1985, Coles & Jones 1991, Neyrinck et al. 2009, Wang et al. 2011)



Our novel statistics

- The crucial features of the late-time matter distribution:
  - the density field is manifested in a hierarchical web-like structure

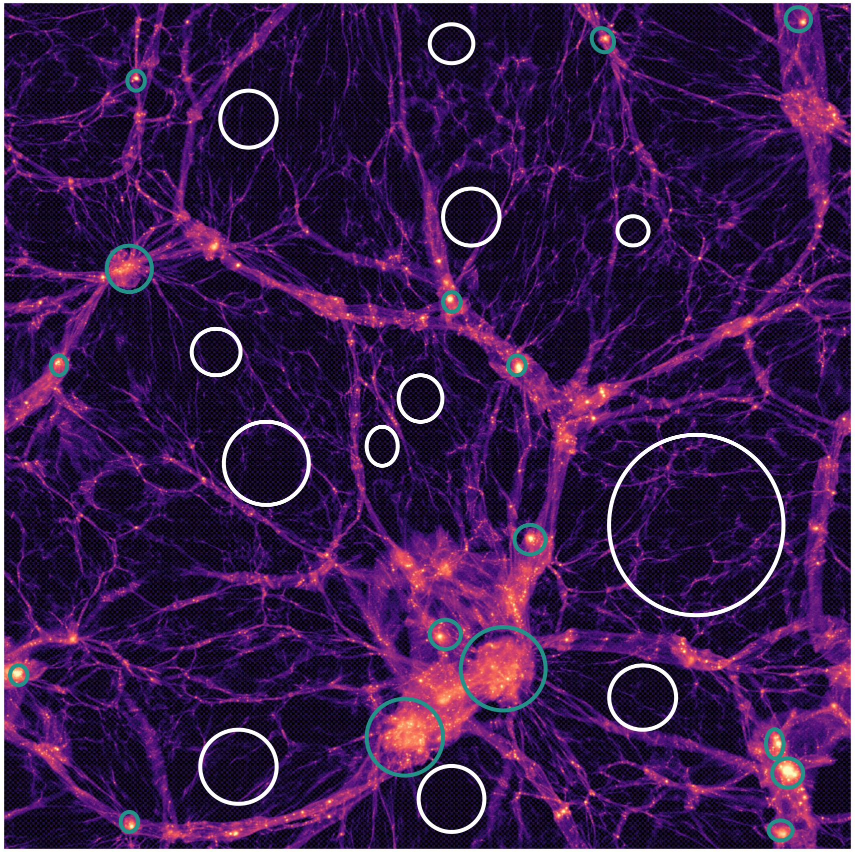
(Sheth & van de Weygaert 2004, Sheth 2004, Shen et al. 2006)



(credit: William A. Watson2014)

Our novel statistics

- The crucial features of the late-time matter distribution:
  - the local extrema of density field are particularly sensitive to the PNG (Dalal et al. 2008, Chan et al. 2019)



Our novel statistics

- Logarithmic transform

$$\rho_{\ln}(\mathbf{x}) = \ln[\rho(\mathbf{x})/\bar{\rho}]$$

- Continuous wavelet transform (CWT)

$$\tilde{\rho}_{\ln}(w,\mathbf{x}) = \int \rho_{\ln}(\mathbf{x}')\Psi(w,\mathbf{x}-\mathbf{x}')\mathrm{d}^3\mathbf{x}'$$

  - the rescaled wavelet: $\Psi(w,\mathbf{x}) = w^{3/2}\Psi(w\mathbf{x})$
  - $w \in \{w_0, w_0+\Delta w, w_0+2\Delta w, \dots, w_0+i\Delta w, \dots\}$

- the isotropic Gaussian-derived wavelet (GDW):

$$\Psi(\mathbf{x}) = C_{\mathrm{N}}(6-|\mathbf{x}|^2)e^{-|\mathbf{x}|^2/4}$$

(Wang & He 2021, Wang et al. 2022)

Our novel statistics

- Detecting peaks (valleys) of $\tilde{\rho}_{\ln}(w, \mathbf{x})$ by locating coordinates with values above (below) their neighbors.
- The scale-dependent peak height function (scale-PKHF) is the number density of CWT peaks with heights falling in the bin $[\nu_{\rm pk} - {\rm d}\nu_{\rm pk}/2, \nu_{\rm pk} + {\rm d}\nu_{\rm pk}/2)$:

$$n_{\rm pk}(w, \nu_{\rm pk}) = \frac{{\rm d}\mathcal{N}_{\rm pk}(w)}{{\rm d}\nu_{\rm pk}}$$

- The scale-dependent valley depth function (scale-VLYDF) is the number density of CWT valleys with depths falling in the bin $[\nu_{\rm vly} - {\rm d}\nu_{\rm vly}/2, \nu_{\rm vly} + {\rm d}\nu_{\rm vly}/2)$:

$$n_{\rm vly}(w, \nu_{\rm vly}) = \frac{{\rm d}\mathcal{N}_{\rm vly}(w)}{{\rm d}\nu_{\rm vly}}$$

- The correspondence: $w = c_w k$, with $c_w = 2/\sqrt{7}$ for the isotropic GDW

(Wang & He 2024)

Parameter forecasts · Fisher analysis

- Parameter vector: $\boldsymbol{\theta} = \{f_{\rm NL}^{\rm local}, f_{\rm NL}^{\rm equil}, f_{\rm NL}^{\rm ortho}, \Omega_m, \Omega_b, \sigma_8, n_s, h\}$
- Statistic vector:

$$\boldsymbol{S} = \Big\{ \begin{array}{llll} n_{\rm vly}(k_0, \nu_{{\rm vly},0}), & n_{\rm vly}(k_0, \nu_{{\rm vly},1}), & n_{\rm vly}(k_0, \nu_{{\rm vly},2}), & \ldots, \\ n_{\rm vly}(k_1, \nu_{{\rm vly},0}), & n_{\rm vly}(k_1, \nu_{{\rm vly},1}), & n_{\rm vly}(k_1, \nu_{{\rm vly},2}), & \ldots, \\ n_{\rm vly}(k_2, \nu_{{\rm vly},0}), & n_{\rm vly}(k_2, \nu_{{\rm vly},1}), & n_{\rm vly}(k_2, \nu_{{\rm vly},2}), & \ldots, \\ n_{\rm pk}(k_0, \nu_{{\rm pk},0}), & n_{\rm pk}(k_0, \nu_{{\rm pk},1}), & n_{\rm pk}(k_0, \nu_{{\rm pk},2}), & \ldots, \\ n_{\rm pk}(k_1, \nu_{{\rm pk},0}), & n_{\rm pk}(k_1, \nu_{{\rm pk},1}), & n_{\rm pk}(k_1, \nu_{{\rm pk},2}), & \ldots, \\ n_{\rm pk}(k_2, \nu_{{\rm pk},0}), & n_{\rm pk}(k_2, \nu_{{\rm pk},1}), & n_{\rm pk}(k_2, \nu_{{\rm pk},2}), & \ldots \\ P(k_0), & P(k_1), & P(k_2), & \ldots, \end{array} \Big\}$$

- The 1-σ marginalized error on parameters:

$$\sigma(\theta_i) \geq \sqrt{(\mathcal{F}^{-1})_{ii}}$$

- Fisher matrix:

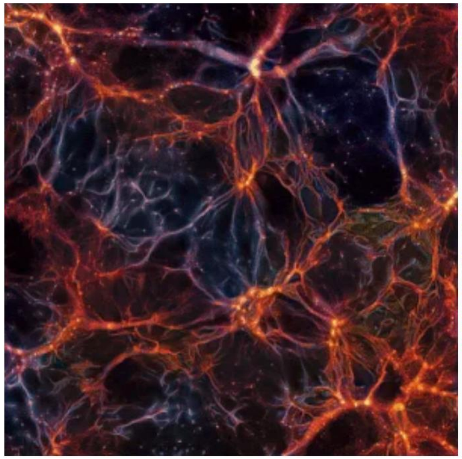
$$\mathcal{F}_{ij} = \left(\frac{\partial \boldsymbol{S}}{\partial \theta_i}\right) \mathcal{C}^{-1} \left(\frac{\partial \boldsymbol{S}}{\partial \theta_j}\right)^T$$

Parameter forecasts · Simulations

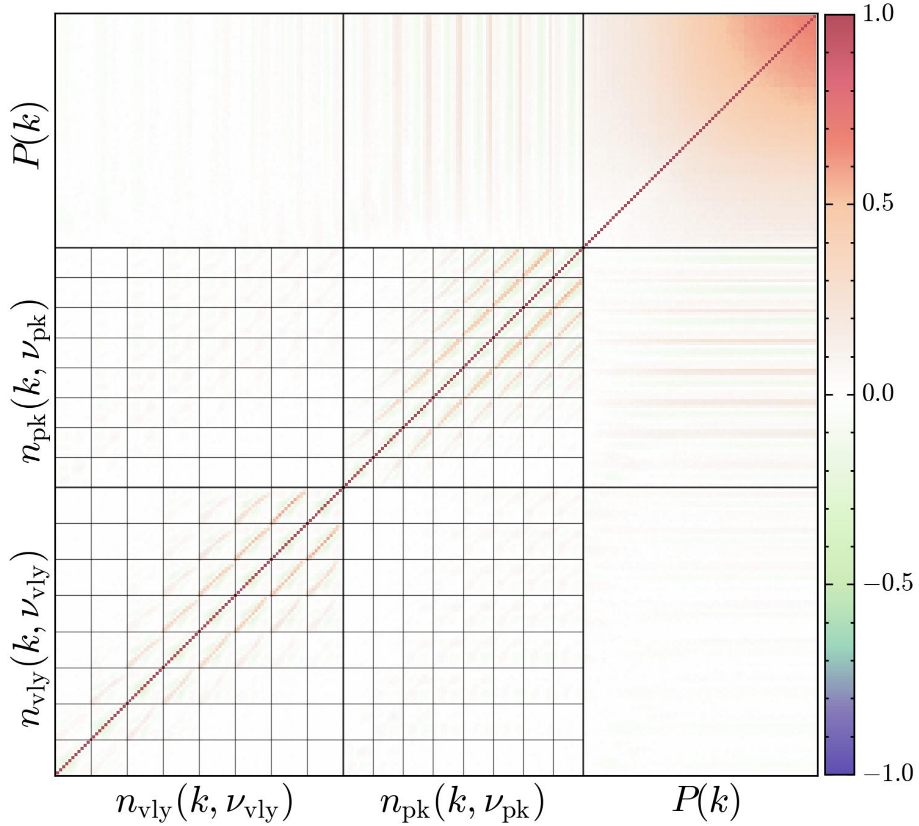
- The Quijote simulation suite: (https://quijote-simulations.readthedocs.io/en/latest/index.html)
  - Goal: quantify the information content on cosmological observables
  - 15000 fiducial simulations with a Planck cosmology

$$\{f_{\rm NL}^{\rm local} = 0, f_{\rm NL}^{\rm equil} = 0, f_{\rm NL}^{\rm ortho} = 0, \Omega_m = 0.3175, \Omega_b = 0.049, \sigma_8 = 0.834, n_s = 0.9624, h = 0.6711\}$$

  - 5 sets of 500 simulations varying one cosmological parameter
  - 3 sets of 500 simulations with $f_{\rm NL} = \pm\ 100$ (Quijote-PNG)
  - Each simulation box contains 512^3 dark matter particles and has a size of 1 Gpc/h




- The correlation matrix $r_{ij} = C_{ij}/\sqrt{C_{ii}C_{jj}}$
- The covariances of the scale-PKHF and scale-VLYDF are more diagonalized
- The scale-PKHF, scale-VLYDF and power spectrum are almost uncorrelated with each other

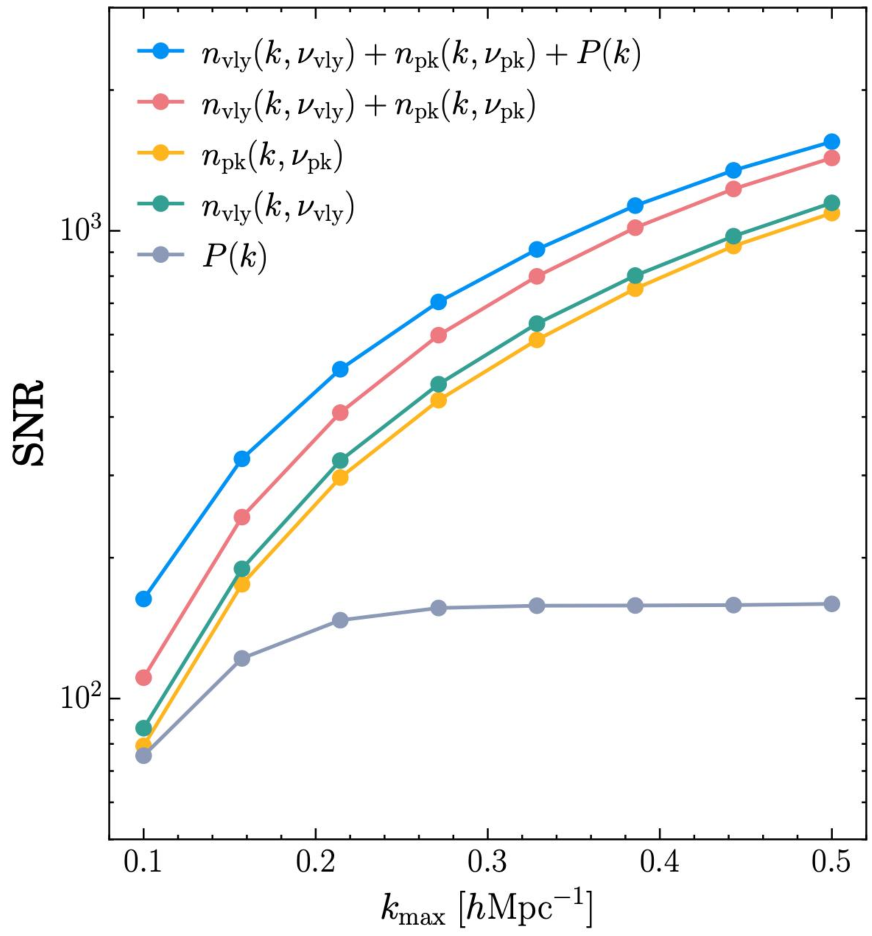


- The cumulative signal-to-noise ratio (SNR): $\mathrm{SNR} = \sqrt{\boldsymbol{S} C^{-1} \boldsymbol{S}^T}$

- The scale-PKHF and scale-VLYDF do not show the flattening effect

- The combination of scale-PKHF and scale-VLYDF much higher SNR, up to 8.98 times than the power spectrun at $k_{\max} = 0.5$ h/Mpc

- even 9.73 times when the power spectrum is included



- Improvement factors of statistics over the power spectrum: σ_P/σ_S
  - $f_{\rm NL}^{local}$: $n_{\rm vly}+n_{\rm pk}+P > n_{\rm vly}+n_{\rm pk} > P+B > n_{\rm vly} > B > n_{\rm pk} > P$
  - $f_{\rm NL}^{\rm equil}$: $n_{\rm vly}+n_{\rm pk}+P > n_{\rm vly}+n_{\rm pk} > P+B > B > n_{\rm vly} > n_{\rm pk} > P$
  - $f_{\rm NL}^{\rm ortho}$: $n_{\rm vly}+n_{\rm pk}+P > n_{\rm vly}+n_{\rm pk} > P+B > B > n_{\rm vly} > n_{\rm pk} > P$
  - Ω_m: $n_{\rm vly}+n_{\rm pk}+P > P+B > B > n_{\rm vly}+n_{\rm pk} > P > n_{\rm vly} > n_{\rm pk}$
  - Ω_b: $n_{\rm vly}+n_{\rm pk}+P > P+B > B > n_{\rm vly}+n_{\rm pk} > n_{\rm vly} > n_{\rm pk} > P$
  - σ_8: $n_{\rm vly}+n_{\rm pk}+P > P+B > B > n_{\rm vly}+n_{\rm pk} > n_{\rm vly} = n_{\rm pk} > P$
  - n_s: $n_{\rm vly}+n_{\rm pk}+P > n_{\rm vly}+n_{\rm pk} > n_{\rm vly} > n_{\rm pk} > P+B > B > P$
  - h: $n_{\rm vly}+n_{\rm pk}+P > P+B > B > n_{\rm vly}+n_{\rm pk} > n_{\rm vly} > n_{\rm pk} > P$

Paras	σ_P/σ_B [19]	σ_P/σ_{P+B} [19]	$\sigma_P/\sigma_{n_{\rm vly}}$	$\sigma_P/\sigma_{n_{\rm pk}}$	$\sigma_P/\sigma_{n_{\rm vly}+n_{\rm pk}}$	$\sigma_P/\sigma_{n_{\rm vly}+n_{\rm pk}+P}$
$f_{\rm NL}^{\rm local}$	28.6	57.6	32.7	20.2	60.2	99.1
$f_{\rm NL}^{\rm equil}$	45.1	53.3	28.1	19.5	54.8	116.0
$f_{\rm NL}^{\rm ortho}$	43.5	74.9	29.8	39.3	104.4	112.4
Ω_m	2.5	5.1	0.8	0.7	1.2	5.9
Ω_b	2.4	3.8	1.2	1.1	1.6	4.0
σ_8	10.1	29.9	4.2	4.2	8.8	48.5
n_s	3.2	7.8	12.4	8.6	15.6	22.8
h	2.6	4.9	1.7	1.5	2.4	6.6

[19] Floss & Meerburg 2024

Summary and Outlook

- We introduce two new summary statistics, the scale-PKHF and scale-VLYDF, for constraining PNG
- The scale-PKHF and scale-VLYDF are capable of capturing a wealth of primordial information about the Universe
- The scale-PKHF and scale-VLYDF are complementary to the traditional power spectrum
- Combining them two with the power spectrum improve constraints on all parameters compared to the bispectrum and power spectrum combination
- Our methodology is well-suited for future surveys
- Further research is required
  - theoretical modeling of the scale-PKHF and scale-VLYDF
  - comparing them with other advanced statistics
  - dealing with redshift space distortions
  - investigating the effects of tracer bias

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Capturing primordial non-Gaussian signatures in the late Universe by multi-scale extrema of the cosmic log-density field

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We construct two new summary statistics, the scale-dependent peak height function (scale-PKHF) and the scale-dependent valley depth function (scale-VLYDF), and forecast their constraining power on PNG amplitudes $\{f_{\rm NL}^{\rm local}, f_{\rm NL}^{\rm equil}, f_{\rm NL}^{\rm ortho}\}$ and standard cosmological parameters based on ten thousands of density fields drawn from QUIJOTE and QUIJOTE-PNG simulations at $z = 0$. With the Fisher analysis, we find that the scale-PKHF and scale-VLYDF are capable of capturing a wealth of primordial information about the Universe. Specifically, the constraint on the scalar spectral index n_s obtained from the scale-VLYDF (scale-PKHF) is 12.4 (8.6) times tighter than that from the power spectrum, and 3.9 (2.7) times tighter than that from the bispectrum. The combination of the two statistics yields constraints on $\{f_{\rm NL}^{\rm local}, f_{\rm NL}^{\rm equil}\}$ similar to those from the bispectrum and power spectrum combination, but provides a 1.4-fold improvement in the constraint on $f_{\rm NL}^{\rm ortho}$. After including the power spectrum, its constraining power well exceeds that of the bispectrum and power spectrum combination by factors of 1.1–2.9 for all parameters.

Welcome any comments!